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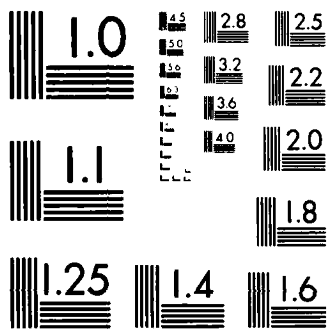

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ESTIMATION BY MOMENTS IN A MODEL OF FAULTY INSPECTION

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ABSTRACT

Methodology developed by Blischke (Ann. Math. Statist. 33 (1962), 444-54) is applied to estimate the parameters in a model of faulty inspection, and to obtain approximate formulae for the variances of these estimators.

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1. INTRODUCTION

Recent papers (Johnson et al. (1980), Johnson & Kotz (1983), Kotz & Johnson (1982)) have developed distributions of observed numbers of apparently defective items when sample inspection is imperfect, resulting in some defectives not being observed as such, and possibly some non-defective being described as 'defective' ("false positives"). Although these results are of interest, some more practical problems arise when it is desired to test whether the inspection is faulty or to estimate the degree of imperfection. In Johnson and Kotz (1985) some tests for detection of faulty inspection were investigated. The present paper is devoted to the estimation aspects of the problem. We will consider here the simplest form of inspection by attributes, assuming lot size to be, effectively, infinite. Each individual in a random sample of size n is examined and a decision reached as to whether or not it is 'nonconforming' (NC). Ideally, of course, such decisions should be completely free of error, but, as is well-known, this is often not the case. As a model of faulty inspection, we introduce two parameters

$$p = \Pr[\text{individual declared NC} \mid \text{individual } \underline{\text{is}} \text{ NC}]$$

$$p' = \Pr[\text{individual declared NC} \mid \text{individual } \underline{\text{is}}\ \underline{\text{not}} \text{ NC}].$$

and suppose we wish to estimate these parameters. The proportion, P, of NC individuals in the lot is unknown, and plays the role, in this context, of a nuisance parameter.

A-1

It is clearly not possible to estimate p and p' (or P) if each individual is examined only once. The only function of the parameters which can be estimated from such data is essentially $Pp + (1-P)p'$ - the probability that an individual chosen at random is declared NC - because the distribution depends only on this quantity.

2. ESTIMATION

If individuals are examined more than once, however, it is possible to estimate each of the three parameters. We will suppose that each of the n individuals in the random sample is examined on m independent occasions. If D_i denote the number of times the i-th individual is declared to be NC, it has the distribution

$$\Pr[D_i=d_i] = P\binom{m}{d_i} p^{d_i}(1-p)^{m-d_i} + (1-P)\binom{m}{d_i} p'^{d_i}(1-p')^{m-d_i} \tag{1}$$

$$(d_i = 0,1,\ldots,m).$$

This is a mixture of two binomial distributions, with parameters (m,p) and (m,p') in proportions P, (1-P) respectively. The r-th factorial moment of each D_i is

$$\mu_{(r)} = E[D_i^{(r)}]$$

$$= E[D_i(D_i-1)\ \ldots\ (D_i-r+1)] = m^{(r)}\{Pp^r + (1-P)p'^r\} \tag{2}$$

Estimating the parameters by making sample and population values of the first three factorial moments agree, we have

$$Pp^r + (1-P)p'^r = F_r \qquad (r=1,2,3) \tag{3}$$

where $F_r = (m^{(r)})^{-1}n^{-1} \sum_{i=1}^{n} D_i^{(r)}$

Solutions $\tilde{p}$, $\tilde{p}'$ and $\tilde{P}$ of (3) are given by Jones (1933) as follows

(a) $\tilde{p}$, $\tilde{p}'$ are roots (in θ) of the equation

$$\theta^2 - A\theta + AF_1 - F_2 = 0 \tag{4}$$

where $A = (F_3-F_1F_2)/(F_2-F_1^2)$ (note that $AF_1-F_2 = (F_1F_3-F_2^2)/(F_2-F_1^2)$)

(b) $\tilde{P} = (F_1-\tilde{p}')/(\tilde{p}-\tilde{p}')$. (5)

There is indeterminacy in the solution, since if $(\tilde{p},\tilde{p}',\tilde{P})$ is a solution, so is $(\tilde{p}',\tilde{p},1-\tilde{P})$. We will adopt the convention of regarding the greater root of (4) as the estimator ($\tilde{p}$) of p. It is reasonable to suppose that p is greater than p' - that is, the probability of declaring an individual to be NC if it is, indeed, NC is greater than if it is not. However, it must be remembered that even if this is so (i.e. $p > p'$), this does not ensure that $\tilde{p}$ must exceed $\tilde{p}'$.

3. Illustrative Example

For purposes of calculation, note that

$$\sum_{i=1}^{n} D_i^{(r)} = \sum_{j=1}^{m} N_j\, j^{(r)} \tag{6}$$

where N_j = number of individuals declared NC on just j occasions among the m times examined.

Suppose we have n=50, m=3; $N_0=43$, $N_1=1$, $N_2=1$, $N_3=5$. Then

$$F_r = (3^{(r)} \cdot 50)^{-1}(43 \cdot 0^{(r)} + 1 \cdot 1^{(r)} + 1 \cdot 2^{(r)} + 5 \cdot 3^{(r)}) ,$$

is
$$F_1 = \frac{1}{150}\,(1+2+15) = \frac{3}{25} ;$$

$$F_2 = \frac{1}{300}\,(2+30) = \frac{8}{75} ;$$

$$F_3 = \frac{1}{300} \cdot 30 = \frac{1}{10} ,$$

where $A = \frac{327}{346} = 0.94508671$ and $\tilde{p}, \tilde{p}'$ are roots of $\theta^2 - 0.945198\theta + 0.00674 = 0$. We find $\tilde{p} = 0.9379$; $\tilde{p}' = 0.0072$; $\tilde{P} = 0.1212$.

In this case $\tilde{p}$, $\tilde{p}'$ and $\tilde{P}$ are all between 0 and 1.

If they are not the method fails, though Blischke (1962, 1964) has suggested rules for this case.

4. Variances

Blischke (1962) has obtained the following asymptotic formulae for the variances of $\tilde{p}$, $\tilde{p}'$ and $\tilde{P}$:

$$\mathrm{var}(\tilde{p}) \simeq \frac{p(1-p)}{P\,nm} + \frac{2(4B_2+B_2')}{nm(m-1)C_2} + \frac{6(B_3+B_3')}{mn(m-1)(m-2)C_4} \qquad (7.1)$$

$$\mathrm{var}(\tilde{p}') \simeq \frac{p'(1-p')}{(1-P)nm} + \frac{2(B_2+4B_2')}{nm(m-1)C_2'} + \frac{6(B_3+B_3')}{nm(m-1)(m-2)C_4'} \qquad (7.2)$$

$$\mathrm{var}(\tilde{P}) \simeq \frac{P(1-P)}{n} + \frac{18(B_2+B_2')}{nm(m-1)(p-p')^4} + \frac{24(B_3+B_3')}{nm(m-1)(m-2)(p-p')^6} \qquad (7.3)$$

where $\quad B_h = Pp^h(1-p)^h; \; B'_h = (1-P)p'^h(1-p')^h$

$$C_h = P^2(p-p')^h; \; C'_h = (1-P)^2(p-p')^h$$

When m is large, the first term of each expression usually gives quite good approximation, so that we may take

$$\text{var}(\tilde{p}) \doteq \frac{p(p-1)}{P\,mn}\;;\; \text{var}(\tilde{p}') \doteq \frac{p'(1-p')}{(1-P)mn}\;;\; \text{var}(\tilde{P}) \doteq \frac{P(1-P)}{n}\,. \tag{8}$$

Note that the denominators are, for $\text{var}(\tilde{p})$, the expected number of examinations of NC individuals; for $\text{var}(\tilde{p}')$, the expected numbers of examinations of conforming individuals; and for $\text{var}(\tilde{P})$ the number of individuals in the sample.

Using the numerical values in the example of Section 3, and inserting the values $\tilde{p}, \tilde{p}', \tilde{P}$ for p, p', P respectively, we find (using (7.1)-(7.3)

$\text{var}(\tilde{p}) \doteq 0.004133$

$\text{var}(\tilde{p}') \doteq 0.0000591$

$\text{var}(\tilde{P}) \doteq 0.002170$

The last two terms in the expressions on the right-hand sides of (7.1)-(7.3) are

0.000885 and 0.000044 for $\text{var}(\tilde{p})$;

0.000004 and 0.000001 for $\text{var}(\tilde{p}')$;

0.000036 and 0.000003 for $\text{var}(\tilde{P})$.

So the use of (8) in this case, at least, would give quite good results, even though m is only 3. (To the same order of approximation the three estimators are uncorrelated.)

5. Confidence Intervals

Blischke (1962) also showed that the asymptotic distributions of the estimators are normal. For P, approximate 100(1−α)% confidence regions can be obtained from the inequality

$$\frac{n(\tilde{P}-P)^2}{P(1-P)} < \lambda^2_{\frac{1}{2}\alpha}$$

where $\Phi(-\lambda_{\frac{1}{2}\alpha}) = \frac{1}{2}\alpha$ and $\Phi(y) = (\sqrt{2\pi})^{-1} \int_{-\infty}^{y} e^{-\frac{1}{2}u^2}\, du$. Taking $\alpha =$ 0.05, so that $\lambda^2_{\frac{1}{2}\alpha} = \lambda^2_{0.025} = 3.8416$ the approximate 95% region for P is

$$50(0.1212-P)^2 < 3.8416\ P(1-P)$$

or equivalently

$$53.8416\ P^2 - 15.9616P + 0.73447 < 0$$

that is $0.057 < P < 0.240$.

Unfortunately we cannot use this method to obtain confidence regions for p and p'. The corresponding region for p (using (8)) would be

$$\frac{P\ mn(\tilde{p}-p)^2}{p(1-p)} < \lambda^2_{\frac{1}{2}\alpha}$$

which cannot be used because P is not known. We might replace P by $\tilde{P}$. This would give an asymptotically correct region.

Using the value $\tilde{P} = 0.1212$, and taking $\alpha = 0.05$, as before, we get the (approximate) 95% confidence regions:

for p: $0.1212 \cdot 150(0.9379-p)^2 < 3.8416\, p(1-p)$,

whence $22.0216\, p^2 - 37.9436p + 15.9922 < 0$

leading to the interval $0.7350 < p < 0.9880$;

for p': $18.18\, (0.0072-p')^2 < 3.8416\, p'(1-p')$

whence $22.0216\, p'^2 - 4.1034\, p' + 0.00094 < 0$

leading to the interval $0.00023 < p' < 0.0650$.

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